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$$=\tfrac{1}{2}\Big(\tan^{-1}\frac{\frac{8}{15}+\frac{36}{77}}{1-\frac{8}{15}\cdot\frac{36}{77}}-\tan^{-1}\tfrac{4}{3}\Big)=\tfrac{1}{2}\big(\tan^{-1}\tfrac{1156}{867}-\tan^{-1}\tfrac{4}{3}\big)$$

$$=\tfrac{1}{2}\big(\tan^{-1}\tfrac{4}{3}-\tan^{-1}\tfrac{4}{3}\big)=0.$$

$4\tan^{-1}\frac{1}{5}$ can be written $\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{5}$ and now the addition formulae can be used at once.

Similarly, for any integral coefficients.

Suppose we are to show that $\cos^{-1}\frac{63}{65}+2\tan^{-1}\frac{1}{5}=\sin^{-1}\frac{3}{5}$.

We proceed as follows: $\cos^{-1}\frac{63}{65}=\tan^{-1}\frac{16}{63}$, $\sin^{-1}\frac{3}{5}=\tan^{-1}\frac{3}{4}$, and now we have to show that $\tan^{-1}\frac{16}{63}+2\tan^{-1}\frac{1}{5}=\tan^{-1}\frac{3}{4}$, which is done at once as in previous example.

It is thus seen that the formulae $\tan^{-1}a\pm\tan^{-1}b=\tan^{-1}\dfrac{a\pm b}{1\mp ab}$ will apply to most if not all of the examples as given in our text-books and that the work is done entirely in the inverse notation.

CALCULUS.

84. Proposed by C. HORNUNG, A. M., Professor of Mathematics, Heidelberg University, Tiffin, Ohio.

Find the equation to the curve upon which a given ellipse must roll in order that one of its foci may describe a straight line.

No solution of this problem has yet been received.

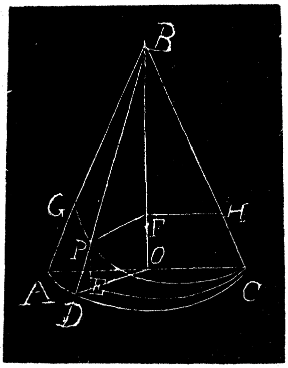
It can be easily shown that $ds=(1+\frac{\rho}{\rho'})ds'$ where ρ and ρ' are the radii of curvature of the rolling and fixed curves, and ds and ds' arcs of the roulette (the curve generated by the focus of the ellipse) and the pedal curve, the origin of which is the generating point. Finding expressions for ds and ds' and substituting in the above equation and solving for ρ, we have an expression for the curvature of the fixed curve. By substituting for ρ, the general value for the radius of curvature, we derive a very complicated differential equation of the curve. I am not aware that the problem has ever been solved. If any of our contributors will send us a complete solution we shall be pleased to publish it. Ed. F.

85. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

A line of double curvature, beginning at some point in the circumference of the base circle of a right cone, winds itself under the constant inclination β to the base circle around the curved surface of the cone. Find its length and that of its projection upon the base circle.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

Let ABC be the cone, CPG the line of curvature, σ its length, CE its projection on base, s the length of this projection, BO the z-axis. Also let $DO=CO=a$, $PF=r$, $\angle DPE$=semi-vertical angle of cone$=\gamma$, $\angle PCD=\beta$. Let P, C be two consecutive points on the curve. Draw PE perpendicular to DO. Then $PC=d\sigma$, $CE=ds$, $PE=dz$, $DE=dr$, $\angle COE=d\theta$, $\angle DPE=\angle DBO=\gamma$.



Then $DE=PD\sin\gamma=PC\sin\gamma\sin\beta$.

$\therefore\ dr=\sin\gamma\sin\beta d\sigma$.

$\therefore\ \sigma=\dfrac{r}{\sin\gamma\sin\beta}=\dfrac{a}{\sin\gamma\sin\beta}$, where $a=r$.

$PE=PD\cos\gamma=PC\cos\gamma\sin\beta$.

$\therefore\ dz=\cos\gamma\sin\beta d\sigma$. $\therefore\ z=\sigma\cos\gamma\sin\beta$.

$CE=\sqrt{(CP^2-PE^2)}$.

$\therefore\ ds=\sqrt{(d\sigma^2-dz^2)}=d\sigma\sqrt{(1-\cos^2\gamma\sin^2\beta)}$.

$\therefore\ s=\sigma\sqrt{(1-\cos^2\gamma\sin^2\beta)}=\dfrac{a\sqrt{(1-\cos^2\gamma\sin^2\beta)}}{\sin\gamma\sin\beta}$

$r.PFH=PH=PC\cos\beta$.

$\therefore\ rd\theta=\cos\beta d\sigma$.

$\therefore\ d\theta=\dfrac{\cos\beta d\sigma}{r}=\dfrac{\cot\beta}{\sin\gamma}.\dfrac{d\sigma}{\sigma}$. $\therefore\ \theta=\dfrac{\cot\beta}{\sin\gamma}\log\sigma$.

$x=r\cos\theta$, $y=r\sin\theta$, $z=\sigma\cos\gamma\sin\beta=r\cot\gamma$ are the equations to the curve. σ and s as given above are the values asked for in the problem.

MECHANICS.

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77. **Proposed by ELMER SCHUYLER, High Bridge, N. J.**

At what elevation must a shell be projected with a velocity of 400 feet that it may range 7500 feet on a plane which descends at an angle of 30°?

I. **Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.; J. SCHEFFER, A. M., Hagerstown, Md., and the PROPOSER.**

Let $OR=7500$ feet$=R$, $V=400$ feet, $\angle POS=e=$angle of elevation, and $\angle ROS=i=30°$.

$\therefore\ PR:OP=\sin(e+i):\cos i$.

But $OP=vt$, and $PR=\frac{1}{2}gt^2$.

$\therefore\ t=2v\sin(e+i)/g\cos i$. $OR:PR=\cos e:\sin(e+i)$.

$\therefore\ R=gt^2\cos e/2\sin(e+i)=2v^2\sin(e+i)\cos e/g\cos^2 i$.

$$\therefore\ \tan e=\frac{v^2\pm\sqrt{(2v^2gR\sin i-g^2R^2\cos^2 i+v^4)}}{gR\cos i}=\frac{160000\pm143348.62575}{208928.62775}.$$

$\therefore\ \tan e=1.451925$ or $.079699$.

$\therefore\ e=55°\ 26'\ 36''$ or $4°\ 33'\ 24''$.